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LAMINAR HEAT TRANSFER IN DIFFUSER FLOW IN A COAXIAL CONICAL CHANNEL IN THE CASE OF BOUNDARY CONDITIONS OF THE FIRST KIND

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Consideration is given to the problem of convective heat transfer in slow diffuser flows in coaxial annular conical channels of constant width. A solution for thermal boundary conditions of the first kind is obtained by the method of separation of variables. The dependence of the temperature on the coordinates is represented in the form of a sum of two infinite series in confluent hypergeometric functions of the transverse coordinate that are multiplied by an exponential dependence on the longitudinal coordinate. The solution is of interest due to its being a superposition of two solutions, each having its own eigenfunctions and eigenvalues. Relations for evaluation of the initial thermal portion in the considered flows are also given.

To select the optimum design and technological parameters of die heads, it is necessary to know the special features of the flow and heat transfer of the melt in the flow elements of the molding equipment. In the extrusion method of production of strands, granules, tubes, films, etc., on the distribution section of the molding equipment the polymer melt flows in a coaxial conical channel formed by the cone of the head and the mandrel [1, 2], where the melt can cool down or warm up. Modern technologies make it possible to maintain different regimes of heat transfer at the boundaries of the channels, but experimental selection of the optimum characteristics of the process requires appreciable means. The construction of numerical models of treatment processes is not always justified either, since in many cases it is possible to obtain adequate relations between the parameters of the processes using analytical solutions. They can serve as test problems in adjusting numerical codes.

In [3, 4], the problem of isothermal flow in coaxial conical channels with different locations of the boundary surfaces is solved, while in [5, 6] a model of the flow and heat transfer in conical gaps with boundary conditions of the third kind is constructed. In this work, we investigate heat transfer in coaxial conical channels with boundary conditions of the first kind for polymer melts that behave like Newtonian fluids [7] within the ranges of the treatment parameters. In [5], it is shown that for flow rates of the liquid or dimensions of the channel of practical interest [3, 4] the Reynolds number is Re $<< 1$, the Nem–Griffith number is Gn $<< 1$, and the Péclet number is Pe > 100. These evaluations make it possible to consider the flow of the melt as a creeping flow [8] and not to take into account in the heat-transfer equation the dissipation heat and to disregard in it the change in the conductive heat flux along the stream as compared to the change in the convective heat flux and ultimately to write in a biconical system of coordinates (Fig. 1) determined by the transformation [9]

$$z' = R \cos \alpha + \mathrm{X} \sin \alpha \, , \tag{1}$$

$$y' = (R \sin \alpha - \mathrm{X} \cos \alpha) \sin \varphi \, , \tag{2}$$

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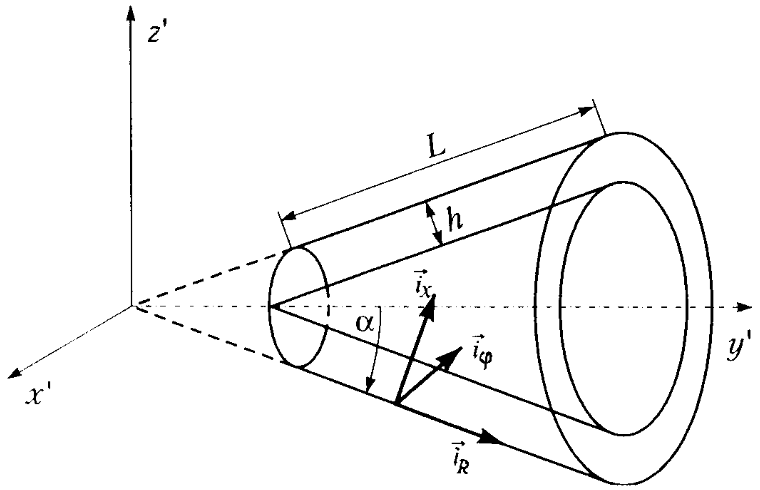

Fig. 1. Geometry of a conical gap of constant width.

$$x' = (R \sin \alpha - X \cos \alpha) \cos \varphi \tag{3}$$

the system of equations of axisymmetric convective heat transfer in the form

$$\frac{\partial \Pi}{\partial \xi} = \frac{1}{\sigma} \frac{\partial}{\partial \chi} \left(\sigma \frac{\partial v}{\partial \chi} \right), \tag{4}$$

$$\frac{\partial \Pi}{\partial \chi} = \frac{\cos(\alpha) \sin(\alpha)}{\sigma^2} v, \tag{5}$$

$$\frac{\partial}{\partial \xi} (\sigma v) = 0, \tag{6}$$

$$\mathrm{Pe}_0 v \frac{\partial \Theta}{\partial \xi} = \frac{1}{\sigma} \frac{\partial}{\partial \chi} \left(\sigma \frac{\partial \Theta}{\partial \chi} \right), \tag{7}$$

where $\xi = R/h$, $\chi = X/h$, $v = V/V_0$, $V_0 = Q/(\pi h(2R_0 \sin \alpha - h \cos \alpha))$, $\Pi = (P - P_0)h/\gamma V_0$, $\sigma = \xi \sin \alpha - \chi \cos \alpha$, $\mathrm{Pe}_0 = V_0 h/a$, $\Theta = (T - T_0)/(T_1 - T_0)$, and T_1 is the temperature of the channel surface formed by the external cone.

The boundary conditions are written in the form

$$v = 0, \quad \chi = 0, \quad \xi_0 < \xi \le \xi_1; \tag{8}$$

$$v = 0, \quad \chi = 1, \quad \xi_0 < \xi \le \xi_1; \tag{9}$$

$$\Pi = 0, \quad 0 \le \chi \le 1, \quad \xi = \xi_0; \tag{10}$$

$$\Theta = 0, \quad 0 \le \chi \le 1, \quad \xi = \xi_0; \tag{11}$$

$$\Theta = 1\,, \quad \chi = 0\,, \quad \xi_0 < \xi \le \xi_1\,; \tag{12}$$

$$\Theta = \Theta_2\,, \quad \chi = 1\,, \quad \xi_0 < \xi \le \xi_1\,. \tag{13}$$

For cases of practical importance where $\xi \tan \alpha >> 1$ the solution of the system of equations (4)–(6) and (8)–(11) in the adopted notation has the form [3]

$$v = \frac{6\,(2\xi_0 \sin\alpha - \cos\alpha)}{\cos\alpha - 2\xi \sin\alpha}\,(\chi^2 - \chi)\,, \tag{14}$$

$$\Pi = -\frac{6\,(\cos\alpha - 2\xi_0 \sin\alpha)}{\sin\alpha} \ln \frac{1 - 2\xi \tan\alpha}{1 - 2\xi_0 \tan\alpha}\,, \tag{15}$$

and then we will write Eq. (7) as

$$\frac{6\mathrm{Pe}_0\,(2\xi_0 \sin\alpha - \cos\alpha)}{\cos\alpha - 2\xi \sin\alpha}\,(\chi^2 - \chi)\;\; \frac{\partial\Theta}{\partial\xi} = \frac{\partial^2\Theta}{\partial\chi^2}\,. \tag{16}$$

The solution of problem (16), (12), and (13) is obtained by the method of expansion in eigenfunctions of the corresponding Sturm–Liouville problem

$$\Theta\,(\xi, \chi) = 1 - (1 - \Theta_2)\,\chi + \frac{1 + \Theta_2}{2} \sum_{n=0}^{\infty} A_{1n} E_{1n}\, {}_1F_1\left(\frac{1 - \mu_{1n}}{4}, \frac{1}{2}; \mu_{1n}\,(2\chi - 1)^2\right) +$$

$$+ \frac{1 + \Theta_2}{2} \sum_{n=0}^{\infty} A_{2n} E_{2n} \sqrt{\mu_{2n}}\,(2\chi - 1)\, {}_1F_1\left(\frac{3 - \mu_{2n}}{4}, \frac{3}{2}; \mu_{2n}\,(2\chi - 1)^2\right), \tag{17}$$

where

$$E_{in}\,(\xi, \chi) = \exp\left\{\frac{8\mu_{in}^2\,(\xi - \xi_0)\,[\cos\alpha - (\xi + \xi_0)\sin\alpha]}{3\mathrm{Pe}} - \frac{\mu_{in}\,(2\chi - 1)^2}{2}\right\}, \quad i = (1, 2)\,;$$

$$A_{1n} = -\frac{\int_{-1}^{1} (1 - t^2)\,\Psi_{1n}(t)\,dt}{\|\Psi_{1n}\|^2}\,; \quad A_{2n} = -\frac{\int_{-1}^{1} t\,(1 - t^2)\,\Psi_{2n}(t)\,dt}{\|\Psi_{2n}\|^2}\,;$$

$$\Psi_{1n} = \exp\left(-\frac{\mu_{1n} t^2}{2}\right) {}_1F_1\left(\frac{1 - \mu_{1n}}{4}, \frac{1}{2}; \mu_{1n} t^2\right) \;\text{ and }\; \Psi_{2n} = \sqrt{\mu_{2n}}\,\times$$

$$\times \exp\left(-\frac{\mu_{2n} t^2}{2}\right) t\; {}_1F_1\left(\frac{3 - \mu_{2n}}{4}, \frac{3}{2}; \mu_{2n} t^2\right)$$

are the eigenfunctions of the Sturm–Liouville operator that corresponds to problem (16), (12), and (13), μ_{1n} and μ_{2n} are the roots of the equations ${}_1F_1\left(\frac{1-\mu}{4}, \frac{1}{2}; \mu\right) = 0$ and ${}_1F_1\left(\frac{3-\mu}{4}, \frac{3}{2}; \mu\right) = 0$, respectively, $\|\Psi_{in}\|^2 =$

$\int_{-1}^{1} (1 - t^2)\, \Psi_{in}^2 dt$ is the square of the norm of the eigenfunctions, and ${}_1F_1(\alpha, \gamma; x)$ is the confluent hypergeometric function. Furthermore, the notation $\mathrm{Pe} = \dfrac{Q}{\pi a h}$ is introduced here, and then $\mathrm{Pe}_0 = \dfrac{\mathrm{Pe}}{2\xi_0 \sin\alpha - \cos\alpha}$.

We note that the value of μ_n can be calculated with an accuracy acceptable for calculations using the relation [10] $\mu_{1n} = 4n + 5/3$, and, as calculations show, the relation $\mu_{2n} \approx \mu_{1n} + 2$ holds.

Using (17), we can calculate the mass-mean temperature of the flow

$$\overline{\Theta} = \frac{2\pi \int_0^1 v\Theta(\xi, \chi)\,(\xi \sin\alpha - \chi \cos\alpha)\, d\chi}{\overline{v}\pi\,(2\xi \sin\alpha - \cos\alpha)} =$$

$$= \frac{12}{\cos\alpha - 2\xi \sin\alpha} \int_0^1 (\chi^2 - \chi)\, \Theta\, (\xi \sin\alpha - \chi \cos\alpha)\, d\chi\,, \tag{18}$$

where

$$\overline{v} = \frac{2\xi_0 \sin\alpha - \cos\alpha}{2\xi \sin\alpha - \cos\alpha} \tag{19}$$

is the average dimensionless velocity.

Let us determine the local dimensionless heat transfer coefficients Nu by referring the heat-transfer coefficients to the difference between the mass-mean temperature of the fluid and the temperature of the wall:

$$\mathrm{Nu}_i = \frac{\alpha_i h}{\lambda} = (-1)^{i-1} \frac{\left.\dfrac{\partial\Theta}{\partial\chi}\right|_{\chi=i-1}}{\overline{\Theta} - \Theta_{i-1}}. \tag{20}$$

In studying the heat transfer we will consider three characteristic cases of assigning temperatures at the boundaries: the first, where the temperatures at the boundaries are equal, i.e., $\Theta_1 = \Theta_2 = 1$; the second, where the dimensionless temperatures are equal in absolute value and opposite in sign, i.e., $\Theta_1 = -\Theta_2 = 1$; the third, where Θ_2 assumes arbitrary values.

Performing differentiation in (20), we obtain

$$\left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=i-1} = (1 - \Theta_2)\,(S_2 - 1) + (-1)^i\,(1 + \Theta_2)\, S_1\,, \tag{21}$$

where

$$S_1 = \sum_{n=0}^{\infty} A_{1n}\, \mu_{1n} E_{1n}\,(\xi, 0)\,(1 - \mu_{1n})\; {}_1F_1\left(\frac{5 - \mu_{1n}}{4}, \frac{3}{2}; \mu_{1n}\right);$$

$$S_2 = \sum_{n=0}^{\infty} A_{2n} \sqrt{\mu_{2n}}\, E_{2n}\,(\xi, 0)\, \frac{\mu_{2n}\,(3 - \mu_{2n})}{3}\; {}_1F_1\left(\frac{7 - \mu_{2n}}{4}, \frac{5}{2}; \mu_{2n}\right);$$

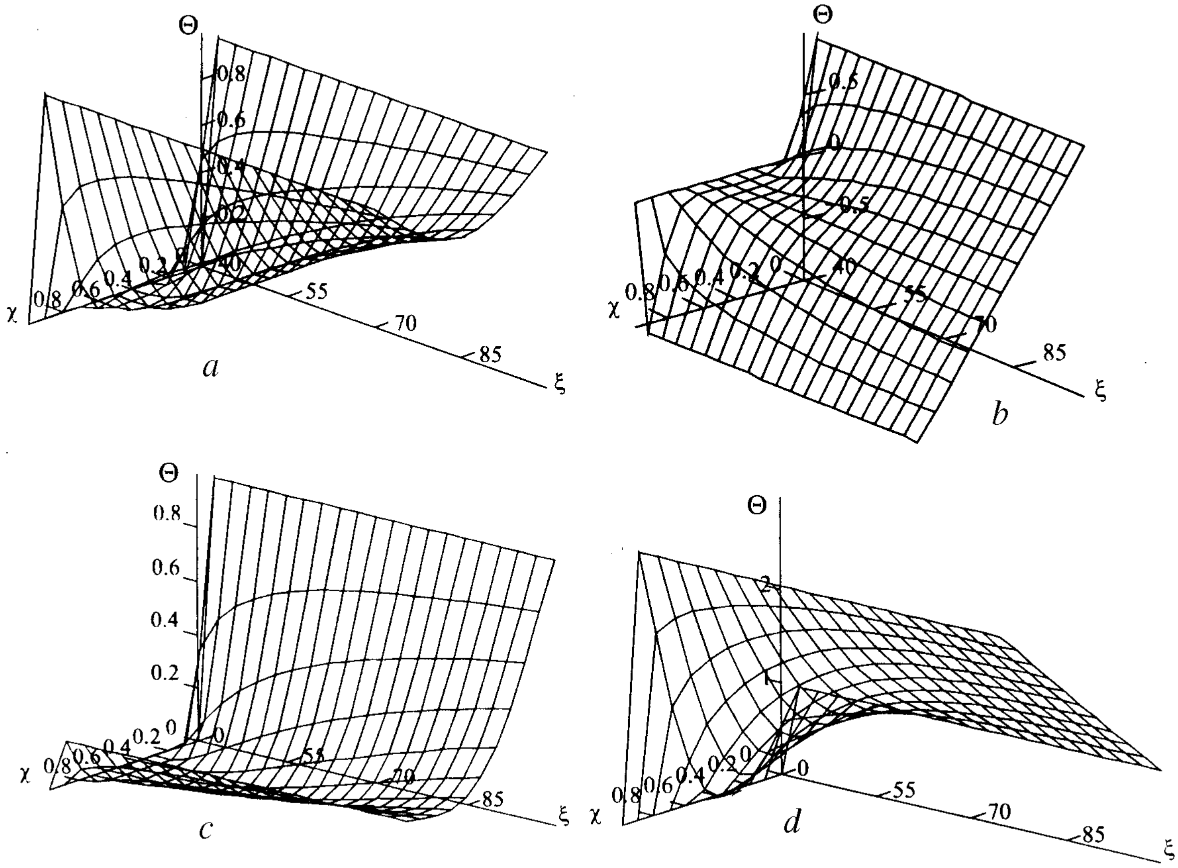

Fig. 2. Distribution of the dimensionless temperature in a channel with the dimensionless parameters $\xi_0 = 40$ and $\xi_1 = 100$: a) for the angle of opening $\alpha = 15^{\circ}$, the Péclet number $\mathrm{Pe} = 10^4$, and the temperatures $\Theta_1 = 1$ and $\theta_2 = 1$ at the channel boundaries; b) for $\alpha = 90^{\circ}$, $\mathrm{Pe} = 7 \cdot 10^4$, $\Theta_1 = 1$, and $\Theta_2 = -1$; c) for $\alpha = 15^{\circ}$, $\mathrm{Pe} = 2 \cdot 10^4$, $\Theta_1 = 1$, and $\Theta_2 = 0.2$; d) for $\alpha = 90^{\circ}$, $\mathrm{Pe} = 10^4$, $\Theta_1 = 1$, and $\Theta_2 = 3$.

$$E_{1n}(\xi, 0) = \exp\left\{\frac{8\mu_{in}^2 (\xi - \xi_0) [\cos\alpha - (\xi + \xi_0)\sin\alpha]}{3\mathrm{Pe}} - \frac{\mu_{in}}{2}\right\}.$$

From (21) it follows that $\mathrm{Nu}_2 = \mathrm{Nu}_1$ for $\Theta_1 = \Theta_2$, and $\mathrm{Nu}_2 = -\mathrm{Nu}_1$ for $\Theta_1 = -\Theta_2$.

We consider the special features of heat transfer in a channel with a dimension $\xi = 40$ and a dimensionless length of the generatrix $\Delta\xi = \xi_1 - \xi_0 = L/h = 60$.

With boundary conditions of even parity ($\Theta_1 = \Theta_2 = 1$) the temperature of the fluid near the walls changes practically in like manner (Fig. 2). For values of the Péclet number $\mathrm{Pe} < 10^4$ the thermal boundary layer spreads to practically the entire cross section of the flow at a short distance from the inlet, and the Nusselt numbers rapidly decrease along the flow, reaching their limiting value $\mathrm{Nu}_\infty = 3.77035$ (Fig. 3), which coincides with the value Nu_∞ for a plane channel – 3.77035 [11]. At large values of the Péclet number $\mathrm{Pe} > 10^4$ the temperature of the fluid does not manage to spread uniformly over the cross section of the flow during the stay of the fluid in the channel, and the Nusselt numbers do not reach their limiting values.

An increase in the angle of the channel's opening α leads to a decrease in the average velocity of the fluid (19) at equal Pr and ξ_0 and an increase in the heat-exchange surface, which causes the temperature of

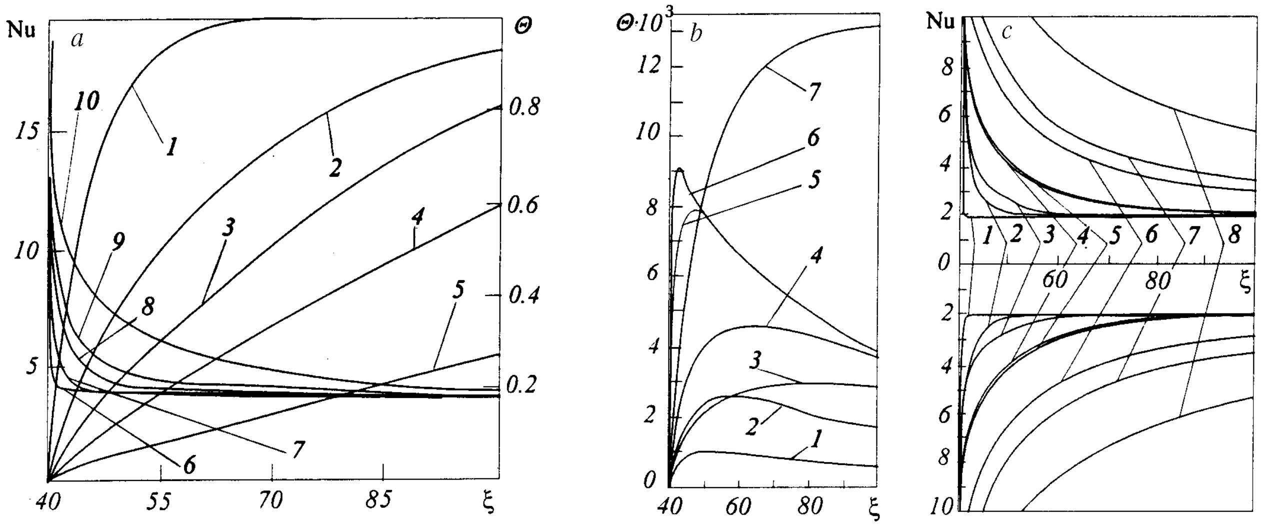

Fig. 3. Nusselt numbers and dimensionless mass-mean temperature of the fluid vs. longitudinal coordinates: a) for a flow in a channel with the temperatures $\Theta_1 = 1$ and $\Theta_2 = 1$ at the boundaries and the angle of opening $\alpha = 15^o$ [1–5) mass-mean temperatures; 6–10) Nusselt numbers; 1, 6) for Pe = 10^3; 2, 7) $2{\cdot}10^3$; 3, 8) 10^{-4}; 4, 9) $7{\cdot}10^4$]; b) mass-mean temperature for a flow with the dimensionless temperatures $\Theta_1 = 1$ and $\Theta_2 = -1$ at the boundaries [1) for Pe = $2{\cdot}10^4$ and $\alpha = 60^o$; 2) $2{\cdot}10^4$ and 30^o; 3) $7{\cdot}10^4$ and 15^o; 4) $2{\cdot}10^4$ and 15^o; 5) $2{\cdot}10^3$ and 15^o; 6) $2{\cdot}10^2$ and 15^o; 7) $2{\cdot}10^4$ and 3^o]; c) for a flow in a channel with the dimensionless temperatures $\Theta_1 = 1$ and $\Theta_2 = -1$ at the boundaries (Nu_1 are positive values and Nu_2 are negative values) [1) for Pe = $2{\cdot}10^2$ and $\alpha = 15^o$; 2) $2{\cdot}10^3$ and 15^o; 3) $2{\cdot}10^4$ and 60^o; 4) $7{\cdot}10^4$ and 90^o; 5) $2{\cdot}10^4$ and 15^o; 6) $7{\cdot}10^4$ and 15^o; 7) $2{\cdot}10^4$ and 3^o; 8) $7{\cdot}10^4$ and 3^o].

the fluid to be equalized more rapidly and the limiting values of the Nu numbers to be reached at a smaller distance (Fig. 3).

Statistical processing of numerical experiments made it possible to obtain an expression for determining the length of the initial thermal portion, i.e., the portion starting with which the Nusselt numbers change by less than 1% in the considered case:

$$l_{\text{in.th}} = (\xi_{\text{in.th}} - \xi_0)\, h = 0.01\, h \sqrt{\xi_0} \left(-\, 3\text{Pe}^{-0.054} + 1.6{\cdot}10^{-1} \frac{\text{Pe}^{0.95}}{\sin \alpha} - \, 4{\cdot}10^{-9} \frac{\text{Pe}^{1.94}}{\sin^2 \alpha} \right). \tag{22}$$

Using (22) one can calculate $l_{\text{in.th}}$ within the following ranges of change in the parameters of the problem with acceptable accuracy: $3 \le \xi_0 \le 500$, $100 \le \text{Pe} \le 10^6$, and $3^o \le \alpha \le 90^o$.

For a flow with boundary conditions of odd parity ($\Theta_1 = -\Theta_2 = 1$) the change in the temperature at the walls of the channel has an opposite character, and at a certain distance from the inlet the dependence of the dimensionless temperature on χ becomes practically linear (Fig. 2), changing from 1 to –1. But due to the difference in the radii of curvature of the inner and outer walls of the channel the average temperature of the fluid differs from 0 and has a nonmonotonic character of change along the stream (Fig. 3).

This is due to the fact that near the inlet to the channel the radii of curvature of the boundary surfaces have the maximum difference and the fluid "heated" near the exterior surface flows in a somewhat greater amount than the fluid "cooled" near the interior surface of the channel. Farther along the flow the

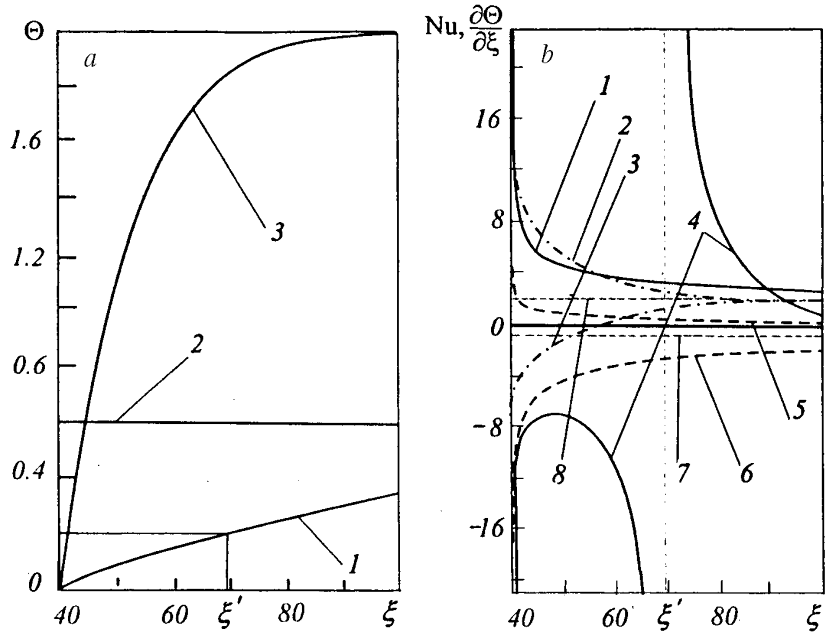

Fig. 4. Dependences on the longitudinal coordinate: a) of the dimensionless mass-mean temperature [1) for a flow in a channel with the angle of opening $\alpha = 15^o$, the Péclet number $Pe = 2\cdot10^4$, and the boundary temperatures $\Theta_1 = 1$ and $\Theta_2 = 0.2$; 2) for steady-state heat transfer in a flow with the parameters $\alpha = 15^o$, $Pe = 2\cdot10^4$, $\Theta_1 = 1$, and $\Theta_2 = 0.2$; 3) for a flow with the parameters $\alpha = 90^o$, $Pe = 10^4$, $\Theta_1 = 1$, and $\Theta_2 = 3$]; b) of the Nusselt numbers (1, 4) and the derivatives $\frac{\partial\Theta}{\partial\chi}$ at the boundaries (2, 3, 5–8) [1) Nu_1 and 4) Nu_2 for a flow in a channel with the angle of opening $\alpha = 15^o$, $Pe = 2\cdot10^4$, and $\Theta_1 = 1$ and $\Theta_2 = 0.2$; 2) $\frac{\partial\Theta}{\partial\chi}\Big|_{\chi=0}$ and 3) $\frac{\partial\Theta}{\partial\chi}\Big|_{\chi=1}$ for the parameters $\alpha = 90^o$, $Pe = 10^4$, and $\Theta_1 = 1$ and $\Theta_2 = 3$; 5) $\frac{\partial\Theta}{\partial\chi}\Big|_{\chi=0}$ and 6) $\frac{\partial\Theta}{\partial\chi}\Big|_{\chi=1}$ for the parameters $\alpha = 15^o$, $Pe = 2\cdot10^4$, and $\Theta_1 = 1$ and $\Theta_2 = 0.2$; 7, 8) limiting values of the derivatives].

difference in the curvature of the surfaces decreases, and the mass-mean temperature will asymptotically approach 0.

Over a certain interval of the development of the temperature field the maximum absolute values of the Nusselt numbers occur close to the inlet, but with the establishment of the temperature distribution the numbera Nu_1 and Nu_2 asymptotically approach their limiting values $Nu_{1\infty} = -2$ and $Nu_{2\infty} = 2$. The increase in the number Pe characterizes the enhancement of the effect of convective heat transfer, which leads to growth of the length of the initial thermal portion (Fig. 3).

The calculations done showed that $l_{in.th}$ is 2–3 times greater with boundary conditions of odd parity than with ones of even parity.

An increase in the angle of opening of the channel α at constant ξ_0 and Pe, just as in the case of heat exchange considered earlier, leads to a decrease in the length of the initial thermal portion. The mass-mean temperature decreases (Fig. 3) due to the decrease in the difference of the radii of curvature of the surfaces forming the channel, and for $\alpha = 90^o$ (radial flow between parallel plates) it becomes equal to 0.

In the case of heat transfer with arbitrarily assigned temperatures at the channel boundaries the main regularities of establishment of the temperature in the channel are consistent with the ones considered earlier, but there are special features in determining the heat fluxes at the channel boundaries.

Since the dimensionless mass-mean temperature of the flow reaches the temperature of one wall of the channel at a certain distance from the inlet ξ (Fig. 4), the Nusselt number determined by the traditional method (20) undergoes a discontinuity on this wall (Fig. 4); at the same time, there are no extreme features in the temperature distribution (see Fig. 2); therefore, in this case expression (20) cannot be used to determine the heat fluxes at the channel boundaries. If in definition (20) the derivative at the boundary is referred to the difference of the dimensionless mass-mean temperature and the larger of the dimensionless temperatures at the boundaries, one of the definitions will lose physical meaning.

If we find the coefficients of heat transfer at the channel boundary as a ratio of the heat flux on the wall to the scale for making the temperature dimensionless, the dimensionless heat flux will be determined as

$$\mathrm{Nu}^* = -\left.\frac{\partial \Theta}{\partial n}\right|_{n=0} , \tag{23}$$

where n is the normal directed toward the fluid, and then we will obtain comparable expressions that determine the dimensionless heat fluxes at the boundaries:

$$\mathrm{Nu}_i^* = (-1)^i \left.\frac{\partial \Theta}{\partial \chi}\right|_{\chi=i-1} . \tag{24}$$

It is clear that expressions (24) reach their limiting values only upon establishment of a practically linear temperature profile (Figs. 3 and 4), and this is true for all the cases considered.

We note that the heat flux at the boundary with a lower dimensionless temperature (if they are of the same sign) changes its direction at a certain distance from the inlet (Figs. 2d and 4), and this distance, as a rule, does not coincide with the distance at which the dimensionless mass-mean temperature reaches the value of the dimensionless temperature at this boundary.

Expressions (24) seem to be the most suitable ones for investigating convective heat transfer in channels whose cross section is multiply connected regions with different temperatures at the boundaries.

In conclusion we note that the functional series in (17) converge uniformly for $\xi > \xi_0$ for parameters of the problem that satisfy the adopted limitations [12]. An analysis of expressions (17) and (21) and numerical summation show that in determining the eigenvalues μ_{in} and calculating confluent hypergeometric functions with a relative error of 10^{-16} the terms of the functional sequences of partial sums of the indicated series starting with the number $n \approx \mathrm{INT}\left[\frac{\sqrt{\mathrm{Pe}}}{(\xi - \xi_0)^2}\right]$, differ by no more than 0.01%. It is with this relative error that the sums of the series in (17) and (21) were calculated. The accuracy of the calculation can be verified using the law of conservation of energy

$$c\rho Q\,(\Delta\overline{T} - T_0) = 2\pi \int\limits_{R_0}^{R_1} [(q_1 - q_2)\, R \sin\alpha + h q_2 \cos\alpha]\, dR , \tag{25}$$

where q_1 and q_2 are the heat fluxes at the boundaries of the channel $\chi = 0$ and $\chi = 1$, respectively. We write (25) in dimensionless form as

$$\mathrm{Pe}\,\overline{\Theta} = 2\left\{\int_{\xi_0}^{\xi}\left[\xi\left(\left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=1} - \left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=0}\right)\sin\alpha - \left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=1}\cos\alpha\right]d\xi\right\}. \tag{26}$$

Calculations show that the right- and left-hand sides of (26) coincide within the accuracy of the computations.

NOTATION

a, thermal-diffusivity coefficient, m^2/sec; *c*, specific heat, J/(kg·K); *h*, channel width, m; $\vec{i}_R$, $\vec{i}_X$, and $\vec{i}_\varphi$, unit vectors in the biconical coordinate system; INT, function of separation of the integral part of a number; *L*, length of the conical portion of the channel, m; *P* and P_0, running and inlet pressures, Pa; *Q*, volume flow rate, m^3/sec; *R*, radial coordinate, m; *T*, temperature, K; *V* and V_0, running velocity and velocity at the inlet to the channel, m/sec; x', y', z', Cartesian coordinates, m; α, half-angle of opening of the cone, rad; γ, dynamic coefficient of viscosity, Pa·sec; λ, thermal conductivity of the fluid, W/(m·K); μ, eigenvalues of the Sturm–Liouville problem; ρ, density, kg/m^3; X, transverse biconical coordinate, m; $\mathrm{Gn} = \frac{\gamma V_0^2}{\lambda \Delta T_{\mathrm{rheol}}}$, Nem–Griffith number; $\mathrm{Pe}_0 = \frac{V_0 h c \rho}{\lambda}$, Péclet number at the inlet to the channel; $\mathrm{Re} = \frac{h V_0 \rho}{\gamma}$, Reynolds number. Subscripts: $i = 1, 2$ (introduced to shorten the representation); *n*, number of the eigenvalue and the corresponding eigenfunction.

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